

Short Papers

Whispering-Gallery Modes in Shielded Hemispherical Dielectric Resonators

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Abstract—The results of the numerical and experimental investigations of whispering-gallery (WG) modes in shielded hemispherical dielectric resonators are presented in this paper. It is shown that the Q factor of WG modes in the shielded resonator can be ten times much higher than the Q factor of the similar open hemispherical dielectric-resonator modes. Shielding the resonator can decrease the dimensions of both the dielectric hemisphere and resonator as a whole, saving the high- Q factor of WG modes. The usage of a cylindrical shield and local flat reflectors in the experiment provides the investigation of the high- Q factor of WG modes in the resonator.

Index Terms—Cylindrical shield, field distribution, Q factor, shielded hemispherical dielectric resonator (SHDR), whispering-gallery (WG) modes.

I. INTRODUCTION

Dielectric resonators (DRs) with whispering-gallery (WG) modes have been under investigation for over 40 years, and they have been especially intensively studied lately [1]–[3]. The reason is the certain advantages of the DRs with WG modes, e.g., the high- Q factor and capability of working in a wide frequency range from the microwave to visible radiation. For the application of a DR in devices, such as a microwave high stable signal source, frequency standard, etc., it is necessary to provide low sensitivity of the DR to environmental perturbations. One of the solutions of this problem is to shield the WG-mode DR. The investigations of the shielded cylindrical DR [4]–[6] are well known. The design of these resonators requires the accurate computation of the resonant frequency and Q factor of the operating mode. However, real definite cylindrical structures are described by only approximate methods [6]. From this point-of-view, spherical structures are very attractive. WG modes in a spherical resonator are higher order modes, of which fields are localized near the resonator boundary between an external and internal caustics. They are separated into two independent families of TM and TE WG modes, called WGE and WGH modes, respectively. The spherical resonators with WG modes can be described by means of rigorous methods and can be used in millimeter-wave and optical techniques, e.g., for frequency stabilization of semiconductor lasers [7] and material studies [8]. On the other hand, there is the problem of a dense spectrum in spherical resonators with WG modes.

Numerical investigation of resonant frequencies of the lower order modes in the dielectric sphere were carried out in [9] in a case without losses, and also in [10] in a case when losses were taken

into account. The frequencies and Q factors of the lower order modes of the dielectric sphere with the ideally conducting spherical shield were considered in [11]. In [3], the resonant frequencies and Q factors of WG modes in an open hemispherical dielectric resonator (OHDR) were given.

In this paper, the results of the investigations of WG modes in a shielded hemispherical dielectric resonator (SHDR) are presented.

II. THEORETICAL FOUNDATION

Let us consider the layered resonator consisting of a dielectric hemisphere (radius r_1 , dielectric permittivity ε_1), hemispherical metal shield of the internal radius r_2 , external radius r_3 , and an ideally conducting flat ground plate. Dielectric media with permittivity ε_2 fills a gap between the shield and dielectric hemisphere. Each dielectric of the resonator is isotropic, homogeneous, and each has a complex dielectric permittivity. The shield has $\varepsilon_3 = i\sigma/(\varepsilon_0\omega_p)$, where σ is the metal conductivity, ε_0 is the permittivity of a vacuum, and ω_p is the resonant frequency of p oscillation. p is the triple index n, m, l , where n, m , and l are the variation numbers along the polar, azimuth, and radial coordinates, respectively. We suppose that the penetration depth of a field is small in comparison with the metal shield thickness because of the high metal conductivity. Independent TM ($H_r = 0$) and TE ($E_r = 0$) modes exist in such a structure.

The solutions of the Maxwell equations satisfying both the condition of finiteness at $r = 0$ and the boundary conditions on the different media interfaces (the continuity conditions of tangential-field components $E_\theta, E_\varphi, H_\theta, H_\varphi$, and $E_\theta = E_\varphi = 0$ at $\theta = \pi/2$) are obtained as a set of spherical functions

$$U_s = \sum_p F_{sp} R_{sp}(r) P_n^m(\cos\theta) \exp[i(m\varphi - \omega t)]. \quad (1)$$

Here, F_{sp} is the mode's amplitude defined from the conditions of excitation, s is the index E for TM modes and H for TE modes. $P_n^m(\cos\theta)$ is the associated Legendre function. The function $R_{sp}(r)$ describes a radial-mode field distribution

$$R_{sp}(r) = \begin{cases} A_s j_n(\sqrt{\varepsilon_1} k_p r), & 0 \leq r \leq r_1 \\ j_n(\sqrt{\varepsilon_2} k_p r) - D_s \eta_n(\sqrt{\varepsilon_2} k_p r), & r_1 \leq r \leq r_2 \\ L_s h_n(\sqrt{\varepsilon_3} k_p r), & r_2 \leq r \leq r_3 \end{cases} \quad (2)$$

where

$$j_n(x) = \sqrt{\frac{\pi x}{2}} J_{n+1/2}(x)$$

$$\eta_n(x) = \sqrt{\frac{\pi x}{2}} N_{n+1/2}(x)$$

$$h_n(x) = \sqrt{\frac{\pi x}{2}} H_{n+1/2}^{(1)}(x); J_\nu(x), N_\nu(x), H_\nu^{(1)}(x)$$

are cylindrical Bessel, Neumann, and Hankel functions of the first kind,

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and $k_p = \omega_p/c$, c is the light velocity in vacuum

$$\begin{aligned}
 A_s &= \frac{\gamma_s}{j_{11}} (j_{21} - D_s \eta_{21}) \\
 D_s &= \frac{j'_{22} h_{32} - \beta_s j_{22} h'_{32}}{\eta'_{22} h_{32} - \beta_s \eta_{22} h'_{32}} \\
 L_s &= \frac{\delta_s}{h_{32}} \left[j_{22} - \eta_{22} \frac{j'_{21} j_{11} - \alpha_s j'_{11} j_{21}}{\eta'_{21} j_{11} - \alpha_s \eta_{21} j'_{11}} \right] \\
 \alpha_H &= \alpha_E^{-1} = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}} \\
 \beta_H &= \beta_E^{-1} = \sqrt{\frac{\varepsilon_3}{\varepsilon_2}} \\
 \delta_E &= \frac{\varepsilon_2}{\varepsilon_3} \\
 \gamma_E &= \frac{\varepsilon_2}{\varepsilon_1} \\
 \delta_H &= \gamma_H = 1 \\
 x_{ij} &= \varepsilon_i k_p r_j \\
 j_{ij} &= j_n(x_{ij}) \\
 h_{ij} &= h_n(x_{ij}) \\
 f'_\nu(x) &= f_{\nu-1}(x) - \frac{\nu f_\nu(x)}{x} \\
 \eta_{ij} &= \eta_n(x_{ij}).
 \end{aligned}$$

The characteristic equations for the modes are given as follows:

$$\begin{aligned}
 & \left(\alpha_s j'_{11} h_{21} - j_{11} h'_{21} \right) \left[\beta_s h'_{32} (j_{22} \eta_{21} - j_{21} \eta_{22}) \right. \\
 & \quad \left. + h_{32} (j_{21} \eta'_{22} - j'_{22} \eta_{21}) \right] \\
 & = -j_{11} (\beta_s h_{22} h'_{32} - h'_{22} h_{32}). \quad (3)
 \end{aligned}$$

At small values of the polar index n , the lower order modes inside the hemisphere possess the volume character. At the large values of a polar index, the higher order modes, i.e., the WGE and WGH modes, travel along the hemisphere boundary and have the field oscillating between an external and inner caustics.

The energy density distribution $W(r)$ for the p th resonant mode can be obtained by averaging energy density distribution $W(r, \theta, \varphi)$ on angular coordinates. We then have the following expression:

$$W(r) = \frac{q_{nm} \rho_s(r)}{2r^2} \left\{ \frac{n(n+1)}{r^2} R_{ns}^2(\chi) + \varepsilon(r) k_p^2 \cdot \left[\left(R'_{ns}(\chi) \right)^2 + R_{n-1,s}(\chi) \right] \right\} \quad (4)$$

where $\rho_E(r) = \varepsilon_0 \varepsilon(r)$; $\rho_H(r) = \mu_0$, μ_0 is the permeability of a vacuum.

The function $R_{ns}(r)$ is obtained by (2), in which χ is equal to $\sqrt{\varepsilon_1} k_p$ at $0 \leq r \leq r_1$; $\sqrt{\varepsilon_2} k_p$ at $r_1 \leq r \leq r_2$; $\sqrt{\varepsilon_3} k_p$ at $r_2 \leq r \leq r_3$, $q_{nm} = 2n(n+1)(n+m)!/(2n+1)(n-m)!$.

III. BACKGROUND OF THE EXPERIMENT

A full spectrum of the SHDR includes higher order radial and azimuth WG modes and other mode types of the structure. The dense spectrum complicates the experimental study and application of the SHDR with WG modes. For overcoming this problem, we propose using a particular SHDR in the experiment, as shown in Fig. 1. In this resonator design, local flat reflectors (1 and 2) are used, on which a dielectric hemisphere (3) and a metal hollow cylinder (4) are placed. The modes in such a partial shielded structure are excited by the open end of the hollow metal waveguide (5) through a coupling slot in the

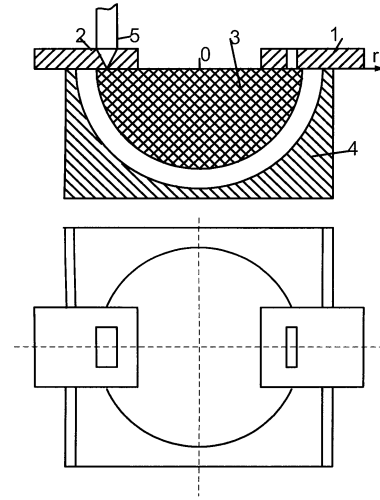


Fig. 1. Schematic view of the shielded resonator design.

reflector (2). The reflector with a coupling slot is able to move along the flat hemisphere surface.

In [3], it was shown that the slot in the OHDR excited the belt type of WG modes. These types of modes exist in the SHDR. The fields of these modes are localized in the narrow area as hemispherical belts between reflectors 1 and 2 (as shown in Fig. 1). The particular shielded resonator does not support higher order radial and azimuth WG modes of the dielectric hemisphere and any modes of the cylinder because of their high radiation losses on the local flat reflectors. The hollow metal cylinder 4 (as shown in Fig. 1), opened with side ends, provides shielding belt type of WG modes and does not support other types of SHDR modes.

Measurement of the Q factor of modes was carried out using an impedance technique [12]. Measurement of the distribution of the resonator-mode fields was carried out by a modified perturbation method. As a small disturbing body, the slot in reflector 1 (as shown in Fig. 1) was used. The displacement of the reflector changed its position in a mode field.

The experimental investigations were carried out in the frequency range of 27–37 GHz. The dielectric hemispheres were made of Teflon and the shields were made of aluminum. The parameters of materials are $\varepsilon_1 = 2.08$, $\varepsilon_2 = 1$, $\tan \delta = 1.8 \times 10^{-4}$, and $\sigma = 3.5 \times 10^7$ S/m. The coupling slot with dimensions $7.2 \text{ mm} \times 0.1 \text{ mm}$ was located in reflector 2, and the disturbing slot with dimensions $7.2 \text{ mm} \times 0.05 \text{ mm}$ was located in reflector 1.

IV. RESULTS AND DISCUSSION

The investigation of the SHDR with different dimensions shows that, by increasing the gap, the resonance frequency of the WGH modes decreases and the resonance frequency of the WGE modes increases, while the unloaded Q factor of both modes gradually increases, exceeding a level of $(\tan \delta)^{-1} = 5.6 \times 10^3$. The resonant frequency of both WGH and WGE modes then becomes practically invariable and equal to the resonant frequency of the corresponding modes of the OHDR [3], and there are the maximum values of the Q factor of both modes. These maximum values of Q depend on the resonant frequency or the magnitude of the polar index n .

The dependency of the unloaded Q factor of the SHDR and OHDR modes for the dielectric hemisphere with $r_1 = 39 \text{ mm}$ on n is shown in Fig. 2. The metal hollow cylinder of radius 42 mm and length 120 mm were used. With the decrease of n , the Q factor of the SHDR mode increases, the Q factor of the OHDR mode drops with the decrease of the index n because of radiation losses. We can see from Fig. 2 that at

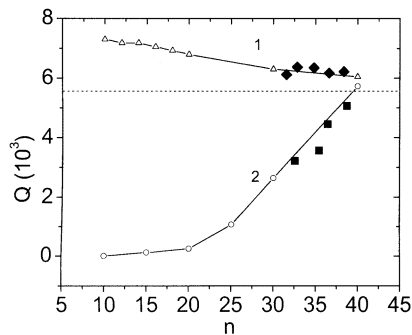


Fig. 2. Q factors of $WGE_{n,1,1}$ modes versus the polar index n . Curve 1 corresponds to the SHDR. Curve 2 corresponds to the OHDR. \blacklozenge and \blacksquare show the experimental results, respectively. The dashed line shows the value of $(\tan \delta)^{-1}$.

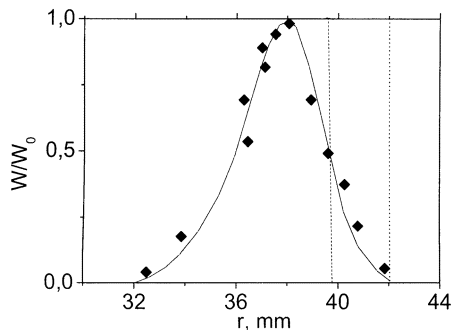


Fig. 3. Radial distribution of the normalized energy density W/W_0 of the $WGE_{30,1,1}$ mode of SHDR. \blacklozenge shows the experimental results. Vertical lines show the radius of the dielectric sphere ($r_1 = 39$ mm) and shield ($r_2 = 42$ mm).

$n < 25$, the Q factor of the WG mode in the shielded resonator can be ten times higher than the Q factor of the similar WG mode of the open resonator.

The increase of the Q factor due to the shielding DR can be explained by wave interference in the gap between the dielectric and metal and, as inquest, the localization of the field of modes in the dielectric and near to it in free space of the gap [4]. It is confirmed by the radial field distribution of one of the WGE modes with $n = 30$ in the shielded resonator, as shown in Fig. 3. As can be seen in Fig. 3, the field of the mode does not reach the metal, and is located in the dielectric and in the air gap near it.

With the decrease of n , the Q factor of the SHDR mode is increased (Fig. 2) because of the increasing part of the mode field in the gap.

Numerical investigation of different kinds of modes shows that with the decrease of n , increasing the Q factor of the WGE mode is greater than increasing the Q factor of the WGH mode. This can be explained as the difference in dependencies of the Q factors' WGH and WGE modes of the SHDR on the polar index in that WGE modes in the dielectric sphere have higher delay factors than WGH modes [3] and their fields penetrate more easily in the gap.

It should be noted that the completely shielded resonator can support different modes of both the structure as a whole and its parts. Thus, there are higher order modes of an inter-spherically resonator formed by spherical surfaces of the shield and dielectric hemisphere. The mode coupling phenomenon occurs at the coincidence of the resonant frequencies of these low- Q -factor modes and the high- Q -factor WG modes. This phenomenon limits the growth of Q (it is not exhibited in Fig. 2) due to the conductor loss in the shield.

The investigation of the dependency of the Q factor on dimensions of the open and shielded resonators shows that the highest level of the Q factor (approximately 5.8×10^3) of the OHDR with $r_1 = 39$ mm can be obtained in the SHDR with $r_1 = 19$ mm and $r_2 = 24$ mm for the WGE mode at $n = 20$ and $m = l = 1$. The total radius of the SHDR in this case is equal to 25 mm for the thickness of the shield of 1 mm (it is quite enough for practical purposes). Thus, shielding the DR can decrease the dimensions of both the dielectric hemisphere and resonator as a whole, saving the high- Q factor of the WG mode.

V. CONCLUSION

SHDRs with the WG modes have been considered in this paper. It has been shown that at particular parameters of the SHDR, the Q factor of its WG modes is ten times more than the Q factor of similar modes in the OHDR. It is physically explained by wave interference in a gap between the dielectric and metal and, as inquest, the localization of the field of modes in the dielectric and near it in free space of the gap. The Q factor of these SHDR modes increases with the decrease of the polar index.

It has also been shown that a shielding DR can decrease the dimensions of both the dielectric hemisphere and resonator as a whole, saving the high- Q factor of WG modes. We suppose that the record significance of the Q factor of WG modes can be reached for the SHDR with high-quality dielectrics at low temperatures and the optimal value of an air-filled gap.

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